$2n$-point renormalized coupling constants in the three-dimensional Ising model: Estimates by high temperature series to order β^{17}

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We compute the $2n$-point renormalized coupling constants in the symmetric phase of the three-dimensional (3D) Ising model on the simple cubic lattice in terms of the high temperature expansions $O(\beta^{17})$ of the Fourier transformed $2n$-point connected correlation functions at zero momentum. Our high temperature estimates of these quantities, which enter into the small field expansion of the effective potential for a 3D scalar field at the infrared fixed point or, equivalently, in the critical equation of state of the 3D Ising model universality class, are compared with recent results obtained by renormalization group methods, strong coupling, stochastic simulations, as well as previous high temperature expansions. [S1063-651X(97)00104-9]

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I. INTRODUCTION

In recent times a considerable effort has been devoted to the evaluation of the $2n$-point dimensionless renormalized coupling constants (RCC's) at zero momentum for the Ising model in three dimensions. These quantities are of interest for constructing the field theoretic effective potential [1,2] of a three-dimensional (3D) scalar field at the infrared fixed point or, in statistical mechanics language, for the formulation of the critical equation of state of the 3D Ising model universality class [3–5]. The computational methods, which so far have been used, include various approximate forms [6–11] of the renormalization group (RG), the field theoretic strong coupling expansion [2], the high temperature (HT) expansion [3,4,12–14], and (single-cluster or multicluster) Monte Carlo techniques [5,15–17].

In this paper we want to discuss how helpful in getting a first estimate of the RCC's in the symmetric phase, can be extensive HT expansion data published long ago [18] and so far only partially analyzed. Indeed expansions as double series in the HT variables $v=\tanh(\beta)$ and $\tau=\exp(\beta H)$, where β is the inverse temperature, are available for the Ising model free energy in a magnetic field H on various two-, three-, and four-dimensional lattices. In particular, in the 3D case the series extend up to order v^{17} for the simple cubic (sc) lattice, up to v^{13} for the bcc lattice and up to v^{10} for the fcc lattice. By computing the $2n$th derivative of the free energy with respect to the magnetic field at zero field we readily obtain the HT expansion of the Fourier transformed $2n$-point connected correlation function at zero momentum (also called the $2n$th susceptibility)

$$\chi_{2n}(v)=\sum_{x_2,x_3,\ldots,x_{2n}}\langle s(0)s(x_2)s(x_3)\cdots s(x_{2n})\rangle_c . \quad (1)$$

These expansions together with that of the second moment correlation length $\xi^2(v)=\mu_2(v)/6\chi_2(v)$ are the essential ingredients for the calculation of the RCC's. The expansion of the second moment of the two-point correlation function $\mu_2(v)$ on the sc lattice has been recently extended in Ref. [19].

In terms of these quantities, the first few RCC's, in the symmetric phase, are defined [2] as the values g_{2n}^+ $(n\geqslant 2)$, that the following expressions:

$$g_4(v)=-\frac{V}{4!}\frac{\chi_4(v)}{\xi^3(v)\chi_2^2(v)},$$

$$g_6(v)=\frac{V^2}{6!}\left(-\frac{\chi_6(v)}{\xi^6(v)\chi_2^3(v)}+10\frac{\chi_4^2(v)}{\xi^6(v)\chi_2^4(v)}\right),$$

$$g_8(v)=\frac{V^3}{8!}\left(-\frac{\chi_8(v)}{\xi^9(v)\chi_2^4(v)}+56\frac{\chi_6(v)\chi_4(v)}{\xi^9(v)\chi_2^5(v)}-280\frac{\chi_4^3(v)}{\xi^9(v)\chi_2^6(v)}\right),$$

$$g_{10}(v)=\frac{V^4}{10!}\left(-\frac{\chi_{10}(v)}{\xi^{12}(v)\chi_2^5(v)}+120\frac{\chi_8(v)\chi_4(v)}{\xi^{12}(v)\chi_2^6(v)}+126\frac{\chi_6^2(v)}{\xi^{12}(v)\chi_2^6(v)}-4620\frac{\chi_6(v)\chi_4^2(v)}{\xi^{12}(v)\chi_2^7(v)}+15\,400\frac{\chi_4^4(v)}{\xi^{12}(v)\chi_2^8(v)}\right)$$

take as $v\uparrow v_c$. The volume V per lattice site has the value 1 for the sc lattice, $4/3\sqrt{3}$ for the bcc lattice, and $1/\sqrt{2}$ for the fcc lattice.

We recall that scaling implies that, as the critical temperature is approached from above, we have $\chi_{2n}\simeq B_{2n}^+(v_c-v)^{-\gamma-(2n-2)\Delta}$, where Δ is the gap exponent. If we also assume the validity of hyperscaling, we have $2\Delta=3\nu+\gamma$ (where ν and γ are the critical exponents of ξ and χ, respectively), so that the RCC's are finite (and universal) quantities. The quantities g_{2n} are expected [20] to be of the form $g_{2n}(v)\simeq g_{2n}^+ + A_{2n}^+(v_c-v)^\theta+\cdots$ as $v\uparrow v_c$,

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where the dominant universal scaling correction exponent θ has the value $\theta = 0.50(2)$ [25] for the 3D Ising model.

By changing in the functions $g_{2n}(v)$ the variable v into $y=\xi^2(v)$, we obtain the strong coupling expansions, through the order y^{17}, of the functions $\gamma_{2n}(y)$ [2] whose values at $y=\infty$ give the RCC's.

Let us add a few comments concerning the HT and the strong coupling series coefficients of the χ_{2n} on the sc lattice that we have tabulated, up to order v^{17}, in the Appendix, together with the coefficients of the second moment of the correlation function $\mu_2(v)$, in order to provide the interested reader with all data we have used, and, thus, make our calculations easily reproducible. The expansion for χ_4 was first computed [21] through v^{17} using the data of Ref. [18], but only recently could we check it against a completely independent linked-cluster computation through the same order [19]. We should only draw attention to a minor misprint in the last two digits of the coefficient of v^{12}, as reported in Ref. [21]. Concerning the strong coupling expansions, we notice that in Ref. [2] $\gamma_6(y)$ has been tabulated, for any space dimension, through order y^{11}, while $\gamma_8(y)$ and $\gamma_{10}(y)$ through order y^7 only. A further significant extension of all these series can still be performed [19]: only then will a complete check against an independent computation be possible for the coefficients tabulated here.

While this work was being completed, we became aware of a related work [14], also devoted to the analysis of the data of Ref. [18], and where also the low temperature side of the critical region is studied. We decided, therefore, to present only the part of our computation, mainly concerning the higher RCC's, which was not already covered by the very thorough discussion of Ref. [14]. In fact the availability of a longer HT expansion of ξ^2 enables us to also study individual RCC's rather than only ratios among them, and, moreover, gives access to the strong coupling expansions.

II. NUMERICAL RESULTS

We shall now present our estimates of the first few RCC's as obtained from either the HT or the strong coupling expansions and discuss various ''biased'' or ''unbiased'' numerical procedures.

In a first and straightforward approach we estimate g_{2n}^+ by evaluating at $v=v_c$ [22–24] near diagonal Padè approximants (PA's) of the quantity $f_{2n}(v)\equiv g_{2n}^{-2/(3n-3)}(v)$ which has a Taylor expansion in v. This procedure is not convenient for extrapolating $g_{10}(v)$, which changes its sign at some $0<v_0<v_c$. In this case we should consider instead the expression $(v/v_c)^6 g_{10}(v)$, which also has a Taylor expansion in v. Thus, (biasing only v_c) we obtain the estimates: $g_4^+=1.03(3)$, $g_6^+=1.93(8)$, $g_8^+=1.53(36)$, $g_{10}^+=-2.0(9)$.

Here, as in the rest of this report, our estimates are given by a suitably weighted average over the results from the approximants using at least 14 series coefficients and the uncertainties are measured, conservatively, only on the basis of the spread of the results obtained from the highest approximants, always allowing also for the (much smaller) effects of the errors in v_c and θ.

It should be noticed that the central estimate of g_4^+ obtained above is slightly, but significantly larger than the well established RG estimate $g_4^+=0.988(4)$ [25].

This discrepancy leads us to investigate whether and to what extent these values are also affected by a ''systematic'' error due to the nonanalytic corrections to scaling which can spoil the convergence properties of the PA's. It has been suggested in Ref. [26] that these corrections can be allowed for, or at least their effects can be significantly reduced, by performing the quadratic mapping

$$v=v_c\left[1-\frac{(1-z)^2}{(1-z/p)^2}\right],$$

with $p=2\sqrt{2}-1$. Essentially the same results are also obtained by using appropriately designed first order differential approximants [27] in which we can bias both v_c and the scaling correction exponent θ. We arrive thus at our final set of estimates

$$g_4^+=0.987(4),\quad g_6^+=1.57(10),$$
$$g_8^+=0.90(10),\quad g_{10}^+=-0.71(35). \tag{2}$$

While the value of g_4^+ is only slightly lowered (and, thereby, closely reconciled with the most accurate RG estimates), the central estimates of the higher g_{2n}^+ are significantly altered and the uncertainties are reduced. Therefore, it appears that our initial very simple numerical approach was rather inadequate and, moreover, we infer that the amplitudes A_{2n}^+ of the scaling correction terms increase with n. Finally, if we notice that the uncertainties of our estimates grow rapidly with the order of the RCC's, it is clear why, with the presently available series, we have to restrict our calculations to the g_{2n}^+ with $n\leqslant 5$.

It is also interesting to study directly other quantities such as, for instance, appropriate ratios of the functions $g_{2n}(v)$ which do not depend on ξ^2 and might be less sensitive to the scaling corrections, as a means to understand better the actual uncertainties of our numerical procedures. We have therefore considered the expression $T_1^+\equiv[g_6(v)/g_4(v)^2]|_{v\uparrow v_c}$ and we have obtained the estimate $T_1^+=1.75(5)$ neglecting the confluent singularity and, otherwise, $T_1^+=1.59(5)$. Analogously, we have also examined $T_2^+\equiv[g_8(v)/g_4(v)^3]|_{v\uparrow v_c}$ and have estimated $T_2^+=1.29(43)$ by the first method and $T_2^+=0.92(13)$ by the second, while for $T_3^+\equiv[g_{10}(v)/g_4(v)^4]|_{v\uparrow v_c}$ we obtain $T_3^+=-0.7(7)$ and $T_3^+=-0.35(20)$, respectively. All estimates of the T^+ are then completely consistent with the corresponding separate estimates of the g_{2n}^+. Notice that the T_i^+ are simply related to the coefficients F_i of the small field expansion of the ''reduced effective potential'' computed in Ref. [10] as follows: $T_1^+=96F_5$, $T_2^+=1728F_7$, and $T_3^+=331\,776/10F_9$.

Let us also recall that long ago the sequence of universal amplitude combinations $I_{2r+3}^+\equiv[\chi_2(v)^r\chi_{2r+4}(v)/\chi_4^{r+1}(v)]|_{v\uparrow v_c}$, $r\geqslant 1$, which are strictly related to the T_i^+, was introduced in Ref. [28] and, by using 12 term series [12], the first few I_i^+ were estimated to be $I_5^+=7.73$, $I_7^+=157.5$, and $I_9^+=6180$ (with no indication of error). Our estimates, by using the direct PA procedure, are

$I_5^+=7.81(3)$, $I_7^+=161.7(3)$, $I_9^+=6395(21)$, while if we allow for the scaling corrections, we find

$$I_5^+=7.92(7), \quad I_7^+=165(4), \quad I_9^+=6809(120). \tag{3}$$

As it appears from the smaller difference between the results of the two kinds of numerical procedures, the T_i^+ and especially the I_i^+ turn out to be less sensitive to the scaling corrections than the g_{2n}^+ and, therefore, we assume that they can be be determined with higher relative accuracy. It is, therefore, interesting to notice that from the above estimate of I_5^+ we get the value $g_6^+=1.62(6)$. Unfortunately, however, at the present level of accuracy, the other simple relations among the higher T_i^+ and the I_i^+, like $T_2^+=\frac{12}{35}(+I_7^+-56I_5^++280)$, etc., which follow from the definitions of the g_{2n}^+, cannot be used for improving the estimates of the higher T_i^+, and, therefore, of the corresponding g_{2n}^+, by expressing them in terms of the I_i^+. For instance, T_2^+ turns out to be a small difference between large numbers and the uncertainty of I_5^+ is strongly amplified. For similar reasons it is also not useful to start directly with the critical amplitudes of the χ_{2n}.

An unbiased study of the RCC's can be performed starting with the strong coupling expansion. In Ref. [2] an elaborate extrapolation procedure was proposed which involves the dependence of the series coefficients on the space dimensionality. We have not yet computed this dependence up to order v^{17} and, therefore, we cannot reproduce this procedure. We can, however, try the simplest approach to evaluate $\gamma_{2n}(\infty)$, which consists in forming $[N+1/N]$ PA's to the quantity $y\gamma_{2n}^{2/(3n-3)}(y)$ and in dividing them by y. This procedure is not very efficient and the only reasonably stable results obtained are $g_4^+=1.1(1)$, $g_6^+=2.1(2)$, $g_8^+=1.9(2)$.

We can also evaluate the ratios T_i^+ by diagonal PA's: again we find reasonable results only for $T_1^+=1.81(4)$. All these values are consistent with our previous first evaluation of these quantities.

Alternatively, we can generalize a technique introduced in Ref. [29], which consists in inverting the functions $z_{2n}=\gamma_{2n}^{-2/(3n-3)}(y)$ (after checking that the dependence of z_{2n} on y is monotonic) and in determining g_{2n}^+ from the value of z_{2n}, where $y=y(z_{2n})$ diverges. This is conveniently done by forming PA's of the logarithmic derivative of y. The results are then $g_4^+=1.01(2)$, $g_6^+=1.63(6)$, $g_8^+=1.05(15)$. As indicated above, these procedures cannot be used for computing g_{10}^+.

In conclusion, we believe that the general consistency among the results obtained by applying suitable approximation procedures to various quantities with somewhat different properties corroborates our estimates in Eq. (2).

III. A COMPARISON WITH OTHER ESTIMATES

Let us now proceed to a comparison with the results already available in the literature. Our values in Eq. (2) for g_4^+ and for g_6^+ are not far from the estimates $g_4^+=1.018(1)$ and $g_6^+=1.793(16)$ obtained in Ref. [14] from the analysis of the same HT series. A similar remark applies to the estimates $g_4^+=0.988(60)$ and $g_6^+=1.92(24)$ obtained in Ref. [13] from a 16 term HT series. Our result for I_5^+ in Eq. (3) is also not far from the recent estimate $I_5^+=7.84(2)$ of Ref. [14]. As to the strong coupling approach, we recall that in Ref. [2] the estimate $g_6^+=1.2(1)$ was obtained from an 11 term strong coupling series.

It is also interesting to perform a comparison with the results obtained in the most extensive recent RG study [10], by the fixed dimension (FD) expansion [25] up to five loop order, resummed by the Borel-Leroy technique combined with an appropriate conformal mapping. The estimate of g_4^+ agrees perfectly with ours, the central values $g_6^+=1.603(6)$ and $g_8^+=0.82(8)$ agree with ours within $\simeq 2\%$ and $\simeq 9\%$, respectively. On the other hand, the larger disagreement about the value of g_{10}^+ should not be taken too seriously because, as noted above, the uncertainty which affects the calculation grows with the order of the RCC. Let us also return to a previous remark, in noticing that from the estimates of F_i in Ref. [25] one arrives at the values $I_5^+=7.945(7)$, $I_7^+=167.45(65)$, and $I_9^+=6718(81)$, in very close agreement with our estimates in Eq. (3). Unfortunately, I_7^+ and I_9^+ are actually rather insensitive to the values of F_7 and F_9.

We also ought to recall that an independent calculation in the FD scheme gave the estimates $g_6^+\simeq 1.50$ in the two loop approximation [7], $g_6^+\simeq 1.622$ at three loop order with a Padè-Borel resummation [8], and $g_6^+\simeq 1.596$ at four loops [9]. On the other hand, from a three loop computation, values for g_8^+ have been obtained [8] which range from 0.68 to 2.71, depending on the resummation procedure.

The approximate truncation of the RG flow equations studied in Ref. [6] yields $g_4^+=1.2$ and $g_6^+=2.25$, which are both clearly larger than our values, although the ratio $g_6^+/g_4^{+2}=1.56$ agrees well with our estimate. An analogous, but lower order truncation of the RG flow equations [11] had given $g_6^+=2.40$.

The $\epsilon=4-d$ expansion approach has not yet been pushed beyond order ϵ^3. It has been used, in Ref. [10], to produce the (rather large) estimates $g_4^+=1.167$, $g_6^+=2.30(5)$, $g_8^+=1.24(8)$, and $g_{10}^+=-1.97(12)$. Notice however that, since also the estimate of g_4^+ is unusually large, the corresponding values of the T_i^+ (or of the F_i) agree very closely with the results from the FD approach. The ϵ expansion of T_i^+ was examined also in Ref. [7], where by a Padè-Borel resummation the estimate $T_i^+=1.653$ was obtained.

Finally, we recall that the Monte Carlo simulations of Ref. [5] indicate $g_6^+=2.05(15)$, which is not very far from our estimate, while the simulations described in Ref. [16] indicate the values $g_6^+=2.7(2)$ and $g_8^+=4.3(6)$, significantly larger than both the RG results and ours. A summary of the present situation is presented in Table I which collects our estimates of the RCC's along with the corresponding ones obtained by other methods.

IV. CONCLUSIONS

We may conclude that although the various computational approaches do not yet agree perfectly, they do appear to converge to common estimates at least for the lowest RCC's. Therefore, in view of the difficulty of these calculations, we

TABLE I. A summary of the estimates of g^+_{2n} by various methods.

Method and Ref.	g^+_4	g^+_6	g^+_8	g^+_{10}
HT	0.987(4)	1.57(10)	0.90(10)	−0.71(35)
Strong coupl.	1.01(1)	1.63(5)	1.05(9)	
HT [14]	1.019(6)	1.791(38)		
HT [13]	0.988(60)	1.92(24)		
RG FD expans. [10]	0.987(2)	1.603(6)	0.83(8)	−1.96(1.26)
RG FD expans. [8,9]		1.596	0.68–2.71	
RG ϵ expans. [10]	1.167	2.30(5)	1.24(8)	−1.97(12)
RG approx. [6]	1.2	2.25		
RG approx. [11]		2.40		
Strong coupl. [2]	0.986(10)	1.2(1)		
MC [5]	0.97(2)	2.05(15)		
MC [16]	1.02	2.7(2)	4.3(6)	

believe that the present residual discrepancies should not be overemphasized. The ϵ expansion is certainly still too short, and perhaps, even for the FD expansions, a further extension would be welcome. The HT series presented here are not yet long enough, the more so the higher the order of the RCC considered. Indeed, we might argue that, at the order v^s, the dominant contributions to the HT expansion of $\chi_{2n}(v)$ come from correlation functions of spins whose average relative distance is $\simeq s/2n$, so that present HT expansions, in some sense, still describe a rather ''small'' system. Analogous problems of size also occur in stochastic simulations [5,16,17]. Therefore further effort would still be welcome to improve the reliability, the precision and, as a result, the consistency of the various approaches.

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APPENDIX: SERIES EXPANSIONS

In the case of the sc lattice the HT expansion of the susceptibilities χ_{2n} are

$$\chi_2(v)=1+6v+30v^2+150v^3+726v^4+3510v^5+16\,710v^6+79\,494v^7+375\,174v^8+1\,769\,686v^9+8\,306\,862v^{10}+38\,975\,286v^{11}+182\,265\,822v^{12}+852\,063\,558v^{13}+3\,973\,784\,886v^{14}+18\,527\,532\,310v^{15}+86\,228\,667\,894v^{16}+401\,225\,368\,086v^{17}+\cdots,$$

$$\chi_4(v)=-2-48v-636v^2-6480v^3-56\,316v^4-441\,360v^5-3\,208\,812v^6-22\,059\,120v^7-145\,118\,844v^8-921\,726\,704v^9-5\,687\,262\,012v^{10}-34\,255\,147\,920v^{11}-202\,130\,397\,708v^{12}-1\,171\,902\,072\,144v^{13}-6\,691\,059\,944\,460v^{14}-37\,693\,869\,995\,312v^{15}-209\,838\,929\,195\,580v^{16}-1\,155\,875\,574\,355\,632v^{17}-\cdots,$$

$$\chi_6(v)=16+816v+19\,920v^2+336\,720v^3+4\,518\,816v^4+51\,745\,680v^5+527\,187\,600v^6+4\,909\,918\,704v^7+42\,581\,232\,864v^8+348\,466\,330\,096v^9+2\,717\,492\,365\,392v^{10}+20\,347\,129\,869\,456v^{11}+147\,133\,138\,147\,872v^{12}+1\,032\,333\,377\,642\,448v^{13}+7\,054\,626\,581\,880\,336v^{14}+47\,100\,223\,055\,946\,160v^{15}+308\,027\,458\,769\,860\,704v^{16}+1\,977\,507\,018\,022\,916\,016v^{17}+\cdots,$$

$$\chi_8(v)=-272-23\,808v-917\,376v^2-23\,013\,120v^3-437\,798\,496v^4-6\,852\,038\,400v^5-92\,654\,596\,992v^6-1\,117\,875\,129\,600v^7-12\,306\,018\,523\,104v^8-125\,633\,562\,017\,024v^9-1\,204\,105\,704\,419\,712v^{10}-10\,936\,791\,715\,557\,120v^{11}-94\,844\,317\,893\,543\,648v^{12}-789\,993\,027\,282\,411\,264v^{13}-6\,351\,007\,395\,937\,161\,600v^{14}-49\,478\,915\,100\,503\,151\,872v^{15}-374\,818\,460\,005\,448\,106\,720v^{16}-2\,768\,750\,733\,973\,561\,834\,752v^{17}-\cdots$$

$$\chi_{10}(v) = +7936 + 1\,061\,376v + 59\,036\,160v^2 + 2\,049\,776\,640v^3 + 52\,252\,083\,456v^4 + 1\,067\,338\,759\,680v^5$$
$$+ 18\,429\,925\,693\,440v^6 + 278\,749\,670\,360\,064v^7 + 3\,786\,553\,881\,275\,904v^8 + 47\,053\,476\,826\,003\,456v^9$$
$$+ 542\,381\,843\,641\,961\,472v^{10} + 5\,862\,580\,439\,606\,155\,776v^{11} + 59\,934\,902\,216\,969\,609\,472v^{12}$$
$$+ 583\,578\,982\,058\,859\,276\,288v^{13} + 5\,442\,873\,762\,995\,091\,611\,136v^{14}$$
$$+ 48\,857\,090\,955\,221\,240\,911\,360v^{15} + 423\,771\,319\,439\,035\,687\,985\,664v^{16}$$
$$+ 3\,563\,795\,335\,882\,672\,497\,655\,296v^{17} + \cdots .$$

The HT expansion of the second moment of the correlation function μ_2 is

$$\mu_2(v) = 6v + 72v^2 + 582v^3 + 4032v^4 + 25\,542v^5 + 153\,000v^6 + 880\,422v^7 + 4\,920\,576v^8 + 26\,879\,670v^9 + 144\,230\,088v^{10}$$
$$+ 762\,587\,910v^{11} + 3\,983\,525\,952v^{12} + 20\,595\,680\,694v^{13} + 105\,558\,845\,736v^{14} + 536\,926\,539\,990v^{15}$$
$$+ 2\,713\,148\,048\,256v^{16} + 13\,630\,071\,574\,614v^{17} \cdots .$$

The strong coupling expansions of the γ_{2n} to order y^{17} are

$$\gamma_4(y) = \frac{y^{-3/2}}{12}[1 + 12y + 6y^2 + 48y^3 - 630y^4 + 7272y^5 - 83\,292y^6 + 957\,312y^7 - 11\,035\,662y^8 + 127\,433\,528y^9$$
$$- 1\,472\,947\,908y^{10} + 17\,036\,529\,504y^{11} - 197\,169\,806\,676y^{12} + 2\,283\,416\,559\,216y^{13}$$
$$- 26\,463\,582\,511\,368y^{14} + 306\,946\,999\,598\,144y^{15} - 3\,563\,327\,123\,879\,550y^{16}$$
$$+ 41\,404\,188\,226\,284\,120y^{17} - \cdots],$$

$$\gamma_6(y) = \frac{y^{-3}}{30}[1 + 18y + 90y^2 + 48y^3 + 576y^4 - 8352y^5 + 114\,528y^6 - 1\,528\,416y^7 + 19\,952\,712y^8 - 255\,983\,472y^9$$
$$+ 3\,240\,722\,592y^{10} - 40\,613\,845\,392y^{11} + 505\,052\,958\,336y^{12} - 6\,242\,882\,909\,472y^{13} + 76\,802\,505\,994\,224y^{14}$$
$$- 941\,288\,338\,072\,752y^{15} + 11\,501\,158\,664\,782\,008y^{16} - 140\,176\,233\,789\,711\,696y^{17} + \cdots],$$

$$\gamma_8(y) = \frac{y^{-9/2}}{56}[1 + 24y + 192y^2 + 576y^3 + 54y^4 + 6720y^5 - 113\,016y^6 + 1\,753\,632y^7 - 25\,771\,326y^8$$
$$+ 364\,798\,032y^9 - 5\,028\,161\,232y^{10} + 67\,958\,735\,808y^{11} - 904\,828\,659\,212y^{12} + 11\,905\,472\,505\,792y^{13}$$
$$- 155\,154\,712\,361\,520y^{14} + 2\,006\,059\,450\,196\,288y^{15} - 25\,765\,180\,820\,314\,374y^{16}$$
$$+ 329\,050\,927\,608\,994\,224y^{17} - \cdots],$$

$$\gamma_{10}(y) = \frac{y^{-6}}{90}[1 + 30y + 330y^2 + 1620y^3 + 3330y^4 - 1080y^5 + 67\,200y^6 - 1\,314\,720y^7 + 22\,683\,000y^8 - 363\,847\,600y^9$$
$$+ 5\,564\,033\,040y^{10} - 82\,249\,187\,520y^{11} + 1\,185\,208\,196\,160y^{12} - 16\,740\,515\,134\,800y^{13}$$
$$+ 232\,658\,153\,938\,560y^{14} - 3\,190\,497\,478\,487\,440y^{15} + 43\,262\,377\,733\,737\,920y^{16}$$
$$- 581\,022\,341\,984\,542\,560y^{17} + \cdots].$$

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